Grade 4: Meaning of Decimals

4.N.9	
Describe and represent decimals (tenths and hundredths) concretely, pictorially, and symbolically.	1. Write the decimal for a concrete or pictorial representation of part of a set (CONCEPTUAL ERROR, not done - see Clarification), part of a region (see Clarification), or part of a unit of measure. 2. Represent a decimal using concrete materials or a pictorial representation. 3. Explain the meaning of each digit in a decimal with all digits the same. 4. Represent a decimal using money values (dimes and pennies). 5. Record a money value using decimals. 6. Provide examples of everyday contexts in which tenths and hundredths are used. 7. Model, using manipulatives or pictures, that a tenth can be expressed as hundredths (e.g., 0.9 is equivalent to 0.90 or 9 dimes is equivalent to 90 pennies).

Clarification of the outcome:

- There are two approaches to teaching decimal meaning: (1) teaching fractions first and then developing decimals by renaming fractions as decimals, (2) developing decimals independently from fractions as an extension of our place value system, and then connecting fractions to decimals (e. g. 3/10 and .3 are different names for the same number). The approach used here is the second one. Why? Because students tend not to understand fractions. Why then connect something not understood to new?
- The part of a set meaning of fraction does not involve equality of parts. Applying the part of a set meaning to decimals can confuse students because equality of parts underlies decimals. Therefore this aspect of achievement indicator #1 is not done in the lesson. The part of a region (part of a whole) sense of fraction is not applied in terms of fraction talk because the lesson does not connect fraction to decimal. It uses measurement to develop decimal meaning.

Required close-to-at-hand prior knowledge:

- Understand length (the concept, and cm and m)
- Understanding place value (ones through thousands)

SET SCENE stage

The problem task to present to students:

Have students play a game. Put a strip of masking tape (or something like it) that is about 2 metres long on the floor. Mark 0 to 4 on the tape (so that there is about a half metre interval between numbers). Mark a throwing line about 1 metre back from the start of the tape. The game involves throwing a sticky ball (such as a small ball of masking tape) on the tape. The object is to have the "ball" land on one of the numbers 0, 1, 2, 3, and 4 that are marked on the tape. Each student has one turn throwing the "ball". If the ball does not land on one of those numbers (the likely event), the student needs to invent a number name for the place the ball landed. The student does this silently. After each student has had a turn, discuss the number names the students invented. Accept all inventions (even fraction names). Do not lead them to a decimal name yet.

Discuss the matter of making a number name for a number between two whole numbers. If a student offers a fraction name, respond by saying something like: "*There are people who don't want fractions. Does this mean that they are not going to be able to name and work with numbers between two whole numbers? Let's see if there is another way to name numbers between two whole numbers*."

Comments:

The purpose of the game and discussion is to have students realize that the number name for the place where the ball landed must somehow be between two whole numbers and that using a fraction name is not the only approach.

In the following lesson, you may be wondering why decimal meaning was developed by using measurement before using money. The reason has to do with the experiences that students have already had with money. They can decode money amounts but that does not mean they understand anything about decimals. In fact, their understanding of money may get in the way of their understanding of decimals if money is used as the initial (and only) teaching context. To put this another way, you want students to realize that decimals are involved in money, but money is not the meaning of decimals.

DEVELOP stage

Activity 1: Addresses achievement indicator 6.

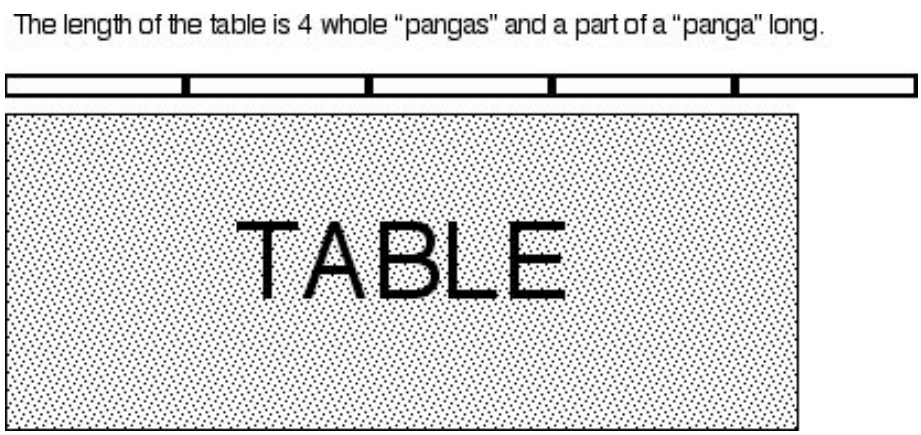
Provide students with magazines, web sites, and newspaper clippings where they can find a dot ('.') used in numbers, in English sentences, and in other situations. Ensure that you include situations where numbers have a dot but the numbers do not involve dollars and cents (e.g. gasoline pumps, times or distances for Olympic athletic events). Have them cut out the pictures/words/situations and make a poster board. Have them briefly describe below each picture/word/situation the purpose of the dot. For example, for a sentence such as 'The lions roared in the night.', they might describe the purpose of the dot as "to show the end of a sentence'. For example, for a number such as $2.35, they might describe the purpose of the dot as "to tell where the dollars and cents are". For example, for '. . .' they might describe the purpose of the dots as "telling that something keeps going". If students cannot describe a purpose for a dot, have them place a large question mark ('?') below the number/situation. [Notice there is a dot at the bottom of a question mark. What is the purpose of the dot?]

Revisit the poster boards and discuss some of the different ways that dots are used to tell about something. Discuss the dot used in money as a way of telling dollars and cents. Then discuss the situation where a dot is used in a number but it does not tell about dollars and cents (e. g. gasoline pump shows 48.7 litres of gas purchased). Ask students if the dot has anything to do with dollars and cents. Present other real situations where a dot in a number does not involve money. Ask students for ideas about what the dot might mean when it is not about dollars and cents. Suggest that using a dot may be a way to name numbers between two whole numbers.

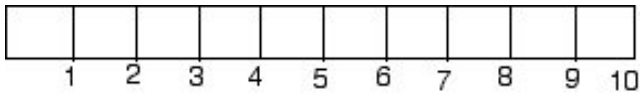
Activity 2: Addresses achievement indicators 1, 2, and 6.

Ask students to invent a name for a measurement unit of length. Note that the invented unit should be at least be as long as someone's foot. [*The invented name used here is 'panga'*.] Ask them to construct an actual panga out of, for example, cardboard.

Ask students to measure the length of a table (for example), using the panga (see diagram). Ensure that the length of the object being measured is at least one whole panga and a part of a panga long. Ask students how we could talk about the part of a whole panga. They might suggest, "*Break the panga into smaller pieces and use them to measure part of a whole panga*."



Discuss their suggestions and eventually steer them in the direction of breaking the unit (e. g. the panga) into ten smaller lengths. Refer to each smaller length as a **ten-bit**. Ask students to mark off the panga into ten-bits and label each ten-bit with a whole number identifier (see diagram). Their ten-bits need not be exactly equal in length; but ensure that they have ten roughly equal ten-bits marked off on the panga.



Ask students to use the panga marked off into ten-bits to remeasure the length of the table. They should now obtain something like 4 whole pangas and 3 ten-bits of a panga long. Ask them to write the length as 43. Ask them if 43 means the table is 43 pangas long. Discuss the situation by reminding them about the purpose of a dot for dollars and cents. Ask if a dot can be used in a similar way to identify whole pangas and ten-bits of a panga. Ask them to write the length, using the money approach and to explain what the dot tells them. Expect: 4.3 pangas long and the dot separates whole pangas from parts of a panga.

Discuss why ten ten-bits of a panga is another name for one whole panga. Connect the measurement situation to place value by comparing how many ten-bits of a panga make a whole panga to how many ones make a ten.

Note:

One way to make sense of the convention that the dot is placed immediately to the right of the ones digit is to compare the situation to the period at the end of a sentence. The period signals the end of a complete thought. The decimal point to the right of the ones digit signals the end of the whole number part of a number.

Activity 3: Addresses achievement indicators 1, 2, and 6. (and practice)

Ask students to use their panga to measure another length in the classroom. Have them record the measurement, using decimal notation (e. g. the width of the book is .8 pangas; the length of the carpet is 7.5 pangas). Repeat about three times. Ask for and discuss the results.

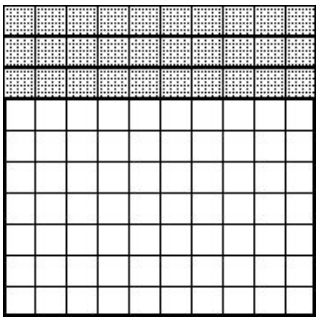
Activity 4: Addresses achievement indicators 1, 2, and 6.

Discuss how a metre and a decimetre are like a panga and a ten-bit of a panga. You will likely have to show students what a decimetre is (1/10 of a metre). If you have a special metre stick that colour codes the decimetres in alternating colours it would help matters go more smoothly. Have students measure a short distance with a metre stick marked off with ten decimetres, using place value notation to write the length (e. g. the table is 1.3 m long). Repeat about three times. Ask for and discuss the results.

Activity 5: Addresses achievement indicators 1, 2, and 6.

Provide students with a hundred grid (that has no numbers in it). Discuss the entire grid represents a whole and that a collection of ten squares (e.g. a row) represents a ten-bit. Ask them to shade in decimal tenths names on the grid (e.g. .3, .9, .2). Ask for and discuss the results.

[**The diagram shows .3.**]



Activity 6: Addresses achievement indicators 1, 2, 4, 5, and 6.

- Write a money value such as $2.3 on the white board (DO NOT write $2.30). Ask students what it might mean. Discuss the situation and ensure that they realize that $2.3 (for example) means 2 whole dollars (loonies) and 3 dimes and that a dollar and a dime is like a panga and ten-bit of a panga, and like a metre and a decimetre. Have them interpret money amounts such as $3.5 and $.7 in terms of whole dollars and dimes. Repeat about four times. Ask for and discuss the results.
- Reverse the task. Provide money amounts (e.g. 4 loonies and 7 dimes.). Ask students to write the money amounts as decimals (e.g. $4.7). Repeat about three times. Ask for and discuss the results.

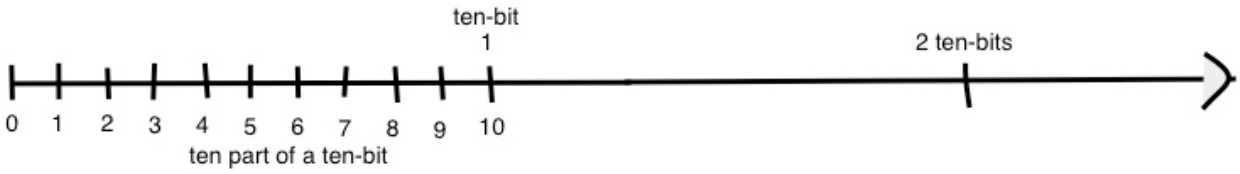
Activity 7: Half-way assessment of teaching.

It is wise to assess your teaching of decimal tenths before developing decimal hundredths.

Ask students to explain how 1.4 pangas, 1.4 m, and $1.4 are similar and how they are different. They should be able to discuss the '.4', in each case, representing 4 parts of a whole unit divided into 10 parts but that the wholes (the units) are different.

Activity 8: Addresses achievement indicators 1, 2, and 6.

Revisit the panga (marked off into ten-bits) and have students use it to measure a length where the length is more than one whole panga long and where the part of a panga is not exact (for example, between 3 ten-bits and 4 ten-bits). Discuss how we might be able to obtain a more accurate measurement for the length. Have students realize (if no one suggests it) that we could subdivide each ten-bit of a panga into ten parts of a ten-bit. Have students subdivide each ten-bit zone into ten parts of a ten-bit (accuracy is not the issue, just ensure they make ten parts of a ten-bit each time). Have them mark number identifiers for the ten parts of a ten-bit but ONLY for the zone between 0 ten-bits and 1 ten-bit. Because of limitations of space, it will likely not be possible to mark each ten part of a ten-bit with a number identifier, but have students mark as many as possible (see magnified diagram where all the ten parts of a ten-bit are identified in the zone 0 ten-bits to 1 ten-bit). Have students provide a more accurate number for the measurement of the length, one that uses ten-bits and ten parts of a ten-bit. See if students are able to write (for example) 2.36 pangas. If not, assist as needed.



Note:

The reason for using the language of ten part of a ten-bit is to help students separate tenths from hundredths through language. Ten-bits are like tenths and ten parts of a ten-bit are like hundredths.

Activity 9: Addresses achievement indicators 1, 2, and 6. (and practice)

Have students use their pangas to measure the lengths of objects in the room, accurate to ten parts of a ten-bit. Have them write the lengths in decimal form (e. g. 1.78 pangas). Repeat about three times. Ask for and discuss the results.

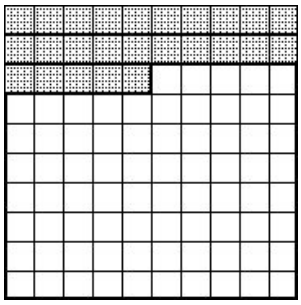
Connect the measurement situation to place value by comparing how many ten parts of a ten-bit make a ten-bit to how many tens make a hundred and by comparing how many ten-bits make a whole panga to how many ones make a ten.

Activity 10: Addresses achievement indicators 1, 2, and 6.

Discuss how a metre, a decimetre (ten centimetres - the 10, 20 , 30 etc. marks on a metre stick), and a centimetre are like a panga, a ten-bit of a panga, and a ten part of a ten-bit. Have students measure a short distance with a metre stick, using place value notation to write the length (e. g. the table is 1.34 m long). Repeat about three times. Ask for and discuss the results.

Activity 11: Addresses achievement indicators 1, 2, and 6.

Provide students with a hundred grid (that has no numbers in it). Discuss the entire grid represents a whole and each small square represents a ten part of a ten-bit. Ask them to shade in decimal hundredths names on the hundreds grid (e.g. .34, .91, .25). [Refer to the example showing .25.] Ask for and discuss the results.



Activity 12: Addresses achievement indicators 1, 2, 4, 5, 6, and 7.

- Put a money value such as $2.36 on the white board. Ask students what it means. Discuss the situation and ensure that they realize that $2.36 (for example) means 2 whole dollars, 3 dimes, and 6 cents and that a dollar, dime, and cent is like a panga, ten-bit, and ten part of a ten-bit. Have them interpret money amounts such as $3.56 and $.73 in terms of whole dollars, dimes, and cents (e. g. $.73 means 0 whole dollars, 7 dimes, and 3 cents). Repeat about four times. Ask for and discuss the results.
- Reverse the task. Provide money amounts (e.g. 4 loonies, 7dimes, 3 pennies.). Ask students to write the money amounts as decimals (e.g. $4.73). Repeat about four times. Ask for and discuss the results.
- Have them convert the dimes and cents to just cents (e. g. 7 dimes and 3 cents is 73 cents). Repeat about four times. Ask for and discuss the results.

Activity 13: Addresses achievement indicators 1, 2, 3, and 6.

Present students with a decimal number where all the digits are the same (3.33, for example). Ask students to explain what each digit refers to in the decimal number. Ask them to draw a diagram and provide a non-money example and a money example that shows the decimal number. Repeat with a different decimal number that has all the digits the same (e.g. 55.55). Ask for and discuss the results.

Activity 14: Addresses achievement indicators 1, 2, 4, 5, 6, and 7. (& practice)

- Present students with a decimal tenth number (e.g. .7). Ask them to show why the decimal number .70 (e.g.) is the same as the decimal number .7, using a hundred grid. Ask them to explain why the .7 (e.g.) is the same as .70 using the context of money. Repeat for two other decimal tenths. Ask for and discuss the results.
- Present students with a decimal hundredth number (e.g. .60). Ask them to show why the decimal number .6 (e.g.) is the same as the decimal number .60, using a hundred grid. Ask them to explain why the .60 (e.g.) is the same as .6 using the context of money. Repeat for two other decimal hundredths. Ask for and discuss the results.

Activity 15: Assessment of teaching

Ask students to explain what each place value position each digit refers to in the number 2.22. Ask them to draw a diagram that shows the decimal number and to provide an example (not money though) that involves the decimal number.

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

Two examples of partially well-designed worksheets follow.

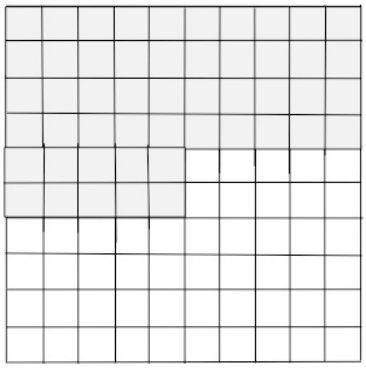
Each worksheet contains a sampling of question types. More questions of each type are needed for a well-designed worksheet.

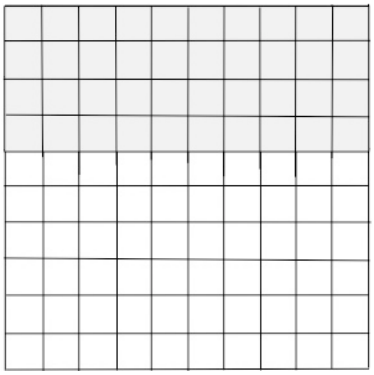
The MAINTAIN stage follows the sample worksheets.

Question 1.

The entire grid represents a whole unit. What decimal number is represented by the shaded part?

_____ _____





Question 2.

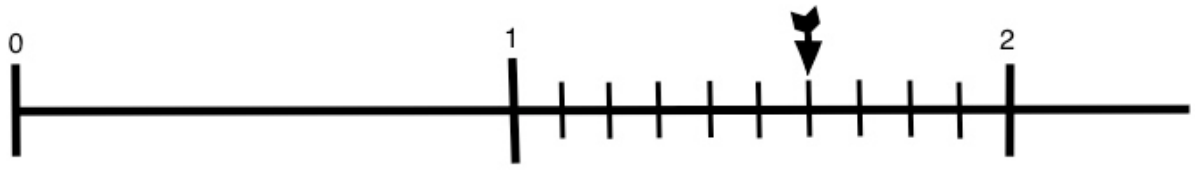
The loonie is the unit.

What decimal number is represented by 2 loonies and 6 dimes? _____

What decimal number is represented by 5 loonies, 6 dimes, and 4 pennies? _____

Question 3.

What decimal number does the arrow point to? _____



Question 4.

The metre is the unit.

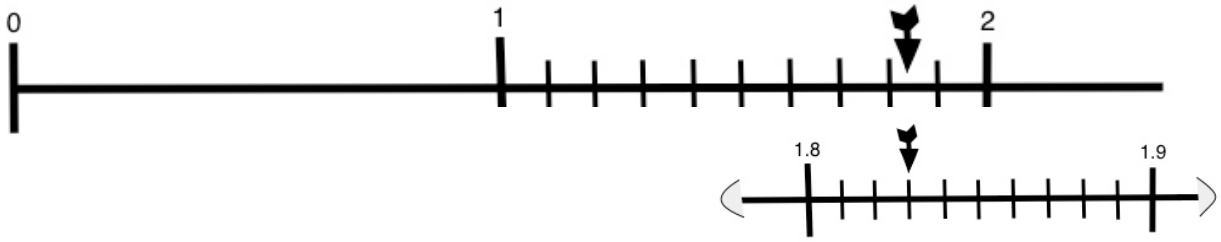
What decimal number is represented by 1 m, 8 dm? _____

What decimal number is represented by 1 m, 8 dm, 9 cm? _____

Question 5.

Look at both rulers. The arrow points to the same decimal number.

What decimal number does the arrow point to? _____



Question 6.

Write the following numbers to show what place value position each digit is in.

Number	Ones	Tenths	Hundredths
7.06			
0.04			
5.70			
0.5			

MAINTAIN stage

Mini-task example

From time to time, write a decimal number (e.g. 1.78) on the board and ask students what each digit refers to (from a place value perspective). Ask students to provide an example of where they might see the number outside of school (other than a money example).

Rich-task example

Provide students with problems that concern money where addition and subtraction of decimal hundredths is required. "Going to the store" contexts work well for this.

Comments.

This is a rich-task because it involves the integration of two mathematics outcomes.